Abstract

In this thesis, we study the parameterized and parallel complexity of hard graph problems for various structural parameters. Our main objective is to study a problem with respect to different structural parameters to obtain a better understanding of the problem's complexity. The problems we consider in this thesis are GRAPH MOTIF, FIREFIGHTING, GRAPH COLORING, HAPPY COLORING, CONFLICT-FREE COLORING and RECOGNITION OF BOUNDED RANK-WIDTH GRAPHS. We study the parameterized complexity of the above problems with respect to several distance-to-triviality parameters. Our notion of distance to a graph class is the vertex deletion distance. More precisely, for a class $\mathcal{F}$ of graphs, we say that the distance of G to the class $\mathcal{F}$ is k if there is a k-sized subset $X \subseteq V(G)$ such that $G \setminus X \in \mathcal{F}$.

- We show that GRAPH MOTIF is fixed-parameter tractable (FPT) when parameterized by the distance to threshold graphs. We give a polynomial kernel for GRAPH COLORING parameterized by the distance to clique.
- We show that FIREFIGHTING is FPT when parameterized by the distance to threshold graphs and distance to disjoint unions of stars. We also show that FIREFIGHTING admits a polynomial kernel when parameterized by the distance to clique, while it is unlikely that it admits a polynomial kernel when parameterized by the distance to disjoint unions of stars.
- We show that MAX HAPPY VERTICES and MAX HAPPY EDGES problems are FPT when parameterized by the treewidth and the number of colors used in the precoloring. Further, we show that both the vertex and edge variants of the problem is FPT when parameterized by either vertex cover number or distance to clique. We also show that the problem of maximizing the number of happy edges is FPT when parameterized by the standard parameter, the number of happy edges in the solution. We show that the MAXIMUM HAPPY VERTEX (EDGE) problem is NP-hard on split graphs and bipartite graphs. We give a polynomial time algorithm to the MAX HAPPY VERTICES problem on cographs.
- We show that CONFLICT-FREE COLORING is FPT when parameterized by the cluster vertex

deletion number of the input graph and admits an additive constant approximation algorithm when parameterized by the distance to threshold graphs. We also study the complexity of the problem on special graph classes, split graphs, cographs and intervals graphs. We also study the conflict-free coloring of points with respect to a set of double intervals. We show that this problem is NP-complete and obtain upper and lower bounds on the number of colors required for the special case when none of the intervals are contained in any other interval.

- We also study about the graph parameter rank-width. We show that for a fixed k, there is an NC algorithm that separates the graphs of rank-width at most k from those with rank-width at least $3k+1$. We also show that some problems which have polynomial time algorithms on bounded rank-width graphs admit NC algorithms on bounded rank-width graphs if a balanced rank-decomposition of bounded width is given as input.