Abstract

Quantum Hall states are robust and good choice for numerous potential applications and to study the physics of topological effects. These states have been observed experimentally in condensed matter systems. However, the observation of fractional quantum Hall states with high flux is difficult in these systems as they require high magnetic fields ($\approx$100 T or more). In this respect, ultracold atoms trapped in the optical lattices are clean and appropriate systems as synthetic magnetic fields equivalent to 1000 T or more can be generated using laser fields. In this thesis, I study the occurrence of quantum Hall states and competing superfluid states in optical lattices for both homogeneous and inhomogeneous systems. And, I also study the physics of multi-component ultracold atomic gases in optical lattices. For the former, I solve the Bose-Hubbard model with synthetic magnetic field referred to as the bosonic Harper-Hofstadter model, using cluster Gutzwiller mean-field and Exact diagonalization methods. The synthetic magnetic field in the optical lattices can be implemented through the Peierls phase and experimentally using laser fields. For the homogeneous case, with the inclusion of the synthetic magnetic field, we obtain the quantum Hall states as the ground state of the bosonic Harper-Hofstadter model. As a first step, the parameters of the QH states are identified based on the compressibility. We obtain the quantum Hall states for different values of synthetic magnetic field with different cluster sizes for the hard core bosons and in the neighbourhood of zero Mott lobe. For the hard core bosons, the onsite interaction energy of the atoms is much larger than the nearest neighbour tunneling energy. As a possible experimental signature, I study in detail the two-point correlation function to distinguish between the quantum Hall states and superfluid states. The states so obtained as further studied in more detail with the exact diagonalization method. I identify the quantum Hall states and superfluid states based on the Penrose-Onsager criterion and Von Neumann entropy. Then, the identification of the quantum Hall states is confirmed by computing the many-body Chern number and ground state degeneracy. For the inhomogeneous case, I do recover all the quantum Hall states for hard wall boundary and for a shallow Gaussian potential, but not with the harmonic oscillator potential. For the multi-component ultracold atomic gases, I obtain the phase diagram for the Bose-Hubbard model for two species. I get the half-filled Mott lobes in presence of inter-species interaction strength. And, show that the width of half-filled Mott lobe varies linearly with increasing the inter-species interaction strength. I, then, consider the nearest neighbour interaction together with onsite interaction in the Bose-Hubbard model and study the phase separation for the strongly correlated phases of two-component ultracold atomic gases in optical lattices. I also study the phase diagram of the extended Bose-Hubbard model and observe the shifting of density wave lobe with the increase of the inter-species interaction strength with zero inter-species nearest neighbour interaction. While with finite inter-species nearest neighbour interaction I observe the phase separation in density wave, supersolid and superfluid phase.

Keywords: Optical lattice, Bose-Hubbard model, quantum Hall states, single-site and cluster Gutzwiller mean-field theory, exact diagonalization.

Work contribution: I was the lead person in defining and developing the ideas of the research projects which form a part of this Ph.D. thesis. I and Soumik contributed equally to the development of the codes and designing the algorithms used. I lead the preparation of the manuscripts of papers published based on the results reported in the thesis.