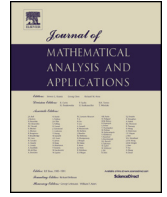




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Multiplicity of solutions to degenerate elliptic problems involving Grushin operator



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ABSTRACT

In this paper, we prove a multiplicity result for solutions to the following class of problems

$$\begin{cases} -\Delta_\gamma v = \lambda|v|^{q-2}v + |v|^{2_\gamma^*-2}v & \text{in } \Omega, \\ v \geq 0 & \text{in } \Omega, \\ v = 0 & \text{on } \partial\Omega. \end{cases}$$

Here, $\Delta_\gamma := \Delta_x + (1+\gamma)^2|x|^\gamma \Delta_y$ is the Grushin operator, $z := (x, y) \in \mathbb{R}^m \times \mathbb{R}^n = \mathbb{R}^N$, $N = m + n$, $m, n \geq 1$, $\gamma > 0$, $\lambda > 0$, $q \in [2, 2_\gamma^*)$, $2_\gamma^* = \frac{2N_\gamma}{N_\gamma - 2}$ is the critical Sobolev exponent, $N_\gamma = m + (1+\gamma)n$, and Ω is a bounded domain of \mathbb{R}^N intersecting the degeneration set of the operator. We employ Lusternik-Schnirelman category theorem to prove the result. More precisely, we show that there exists $\lambda^* > 0$ such that for each $\lambda \in (0, \lambda^*)$, the above problem has at least $\text{cat}_\Omega(\Omega)$ distinct solutions.

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