ABSTRACT

In this study, a new formulation to the fully implicit Roe scheme using the backward Euler method is proposed for the numerical solution of 1-D Euler equations. The implicit fluxes are linearised by implicit local linearisation technique by using the cell face values rather than cell centered values. This time linearised implicit fluxes contain explicit fluxes, jacobians and implicit vector of conserved variables, all at the faces of the cell. The implicit term is an inner product of the flux Jacobian and the vector of difference between the current and previous time-step values of the conserved variables. The Roe's linearization process is applied to the time linearized fluxes to convert the nonlinear jacobian matrices at the faces to constant jacobian matrices and also to evaluate the explicit fluxes. The values of the conserved variables to the left and right side of the cell face are obtained from a one parameter family of Vanleer's Monotone Upwind Scheme for Conservation laws (MUSCL) method which uses linear or parabolic interpolation based on the chosen parameter. The difference between these left and right states is projected on to the eigenvectors from which the expression for implicit vector of conserved variables is obtained, this expression is further tweaked to improve the diagonal dominance of the pentadiagonal system of block matrices. The MUSCL reconstruction is implemented with various limiter functions to ensure that the least amount of numerical viscosity is present in the numerical solution. The use of limiters has an adverse effect on the convergence of the pentadiagonal system of block matrices and is handled by the use of under-relaxation factor. The implicit scheme is applied to various cases of the 1-D shock tube problem. It is observed that the implicit Roe scheme has excess numerical viscosity compared to the existing explicit Roe scheme. The reason for this excess numerical viscosity is

explained from the modified equations to the scalar 1-D advection equation clearly demonstrating that the excess numerical viscosity is inherent to the implicit Roe schemes. It is because of this excess numerical viscosity the implicit Roe scheme is able to overcome the Courant-Friedrichs-Lewy (CFL) time step restriction and go for arbitrarily large time steps. The entropy fix is applied to the implicit scheme by only modifying the eigenvalues in the explicit flux terms. The new implicit scheme helps achieve stable solutions at CFL numbers significantly greater than unity but there exists a case-specific upper limit on the CFL number that can be employed.